Flavour issues for string-motivated heavy scalar spectra with a low gluino mass: the G_2-MSSM case

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Abstract

In recent years it has been learned that scalar superpartner masses and trilinear couplings should *both* generically be larger than about 20 TeV at the short distance string scale if our world is described by a compactified string or M-theory with supersymmetry breaking and stabilized moduli [1]. Here we study implications of this, somewhat generally and also in detail for a particular realization (compactification of M-theory on a G_2 manifold) where there is significant knowledge of the superpotential and gauge kinetic function, and a light gluino. In a certain sense this yields an ultraviolet completion of minimal flavour violation. Flavour violation stems from off-diagonal and non-universal diagonal elements of scalar mass matrices and trilinear couplings, and from renormalization group running. We also examine stability bounds on the scalar potential. While heavy scalars alone do not guarantee the absence of flavour problems, our studies show that models with heavy scalars and light gluinos can be free from such problems.

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1 Introduction

Flavour physics has usually been treated as a low-scale effective theory, ignoring high-scale theories, except perhaps for motivating insights from short-distance physics. It has long been thought by some that high-scale theories with gravity-mediated supersymmetry (SUSY) breaking would typically imply too large flavour-changing neutral current (FCNC) contributions. Some people have argued for heavy scalars to suppress the FCNCs (e.g., [2, 3]), but these are done in the context of the low-scale effective theory, depend on a number of detailed assumptions, and lack deeper motivation. Obtaining detailed flavour predictions from high-scale string theories is difficult because it requires extensive knowledge about the superpotential and the Kähler potential that is not yet available [4].

In recent years there has been progress in constructing string/M-theories compactified to 4D, with broken SUSY and moduli stabilized (as is necessary for any theory to be a candidate to describe our world [1]). Generically such theories have moduli to describe the sizes, shapes, and other properties of the curled up dimensions. The moduli quanta are unstable and decay via gravitational coupling to all matter. If they decayed too late, the successes of big bang nucleosynthesis would be spoiled, and/or they would carry too much energy density. Their lifetime depends on their mass, so consistency with cosmology generically requires that they have masses heavier than about 20 TeV [5].

The importance of this for flavour physics arises because in generic string/M-theories one can show that the gravitino mass, which measures the effects of SUSY breaking, and to which the soft-breaking Lagrangian is proportional, must then itself be heavier than about 20 TeV. Then the supergravity theory implies that the scalar superpartner (squark and slepton) masses, heavy Higgs masses and also the trilinear couplings must all be larger than about 20 TeV as well. That in turn has major effects on flavour physics because the heavy particles and trilinears mainly decouple, though care is needed when the off-diagonal flavour structure is included. Studying these implications is the main goal of this paper.

The results just stated being the generic properties of string/M-theories, if our world is indeed described by a compactified string/M-theory with SUSY breaking and stabilized moduli (which is crucial to define the coupling and masses needed for predictions), it would be rather likely that the world is described by a theory with heavy scalars and trilinears.

If we specialize to M-theory compactified on a manifold with G_2 symmetry, some stronger results hold that may or may not be valid for all string theories. In particular in the M-theory case it has been possible to show that the soft CP-violating phases are zero [6], so there is no weak CP problem, and also that the strong CP problem can be solved [7]. For the G_2 compactification, once the requirement of a de Sitter vacuum and the small cosmological constant are imposed, the gaugino masses are suppressed [8].

With this perspective, we investigate limits from flavour and CP violation on the trilinear couplings and soft-squared scalar masses in supersymmetric models with light gluinos and heavy scalars. Some analyses of this kind have been considered previously [9, 10, 11], focusing on very particular examples, perhaps with only two heavy families, and ad-hoc assumptions for Yukawa and trilinear couplings. One of the reasons for doing this is that a general analysis represents a formidable task without a priori definite information about the form of the Yukawa couplings and the supersymmetric spectrum. However, for a particular set-up consistent with requirements of compactified string theories, we can make some general statements and obtain precise bounds, even though we do not yet know the

precise form of Yukawa couplings in the G_2-MSSM models. Our analysis is organized as follows.

1. While we concentrate on the G_2-MSSM, we analyze three different cases:

   (a) The case where trilinear terms are proportional to Yukawa couplings at the unification scale M_{G}. Such a proportionality is often used in the literature for simplicity. It arises if we have a trivial Kähler potential, which respects the G_2 holonomy, and Yukawa couplings that do not depend on hidden-sector fields. Due to the running of the parameters, Yukawa and trilinear couplings are not proportional to each other at low energies. Hence, non-zero off-diagonal elements in the trilinear matrices remain even after diagonalizing the Yukawa matrices at the electroweak scale.

   (b) The case where trilinear terms are not proportional to Yukawa couplings at M_{G}, but such that the non-proportionality is determined by real factors. Thus, complex phases at high energy enter only via the Yukawa couplings. This should generically happen in the context of the G_2-MSSM [6]. This case can be reproduced with a non-trivial Kähler potential. We generate a series of random numbers determining the non-proportionality.

   (c) The case where trilinear terms are not proportional to Yukawa couplings at M_{G} and where new phases appear at the high scale. This will represent a scenario beyond the G_2-MSSM. We explore this scenario as a contrast to the G_2-MSSM case so that we can determine whether or not there could be an important impact of the phases. Again, we use random numbers determining the non-proportionality, which are now complex.

   We assess the impact of the trilinear terms on flavour and CP violation for each case.

2. At M_{G} the boundary conditions are as follows: the Yukawa matrices of both up- and down-type quarks are non-diagonal complex 3×3 matrices. Their diagonalizing matrices are similar to the CKM matrix V_{CKM}, so their off-diagonal elements are small, except for the right-handed diagonalizing matrix of down quarks, U_R^d, which has sizable off-diagonal elements. For concreteness, we use Yukawa matrices constructed in a grand unified model with a family symmetry [12]. It is important to mention that we have taken this as a definite example but this Yukawa pattern can be embedded in other contexts. The trilinear terms are also non-diagonal complex 3×3 matrices, either (a) proportional or (b) not proportional to the Yukawa matrices without new phases or (c) not proportional to the Yukawa matrices with new CP phases. The soft-squared mass matrices are proportional to the unit matrix at M_{G}. Recall that whenever we have a trivial Kähler metric, the soft-squared masses at that scale will be proportional to the unit matrix, because the same matrices diagonalizing the Kähler metric will diagonalize the soft-squared matrices. Non-trivial Kähler metrics could also reproduce diagonal soft-squared masses but lift the universality condition. As long as $m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2 \lesssim 1.5 \, m_0^2$ at the GUT scale, the result from this analysis will be valid.[1]

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[1]If this inequality is violated, there arises too large an off-diagonal term in the super-CKM basis.

3. We focus our studies on supersymmetric mass spectra featuring heavy scalars ($m_{\tilde{q}} \gtrsim 20$ TeV) and light gauginos. In particular, the light gluino ($m_{\tilde{g}} \lesssim 1$ TeV), due to its strong interactions, can potentially play a significant role for low-energy observables even if the scalars are heavy. For the specification of such SUSY mass spectra, we use the G_2-MSSM [8, 13, 14] as a concrete UV-complete model, which helps us to clarify the potential effects of the high-energy physics on the flavour physics phenomena at the electroweak scale. The model is based on the effective field theories arising from a class of $\mathcal{N} = 1$ fluxless compactifications of M-theory on a G_2 manifold. For concreteness, we choose a set of benchmark G_2-MSSM spectra that has been analyzed in [14].

4. In addition to bounds coming mainly from the kaon sector, we also consider constraints from the stability of the scalar potential, which are relevant for heavy spectra since they are independent of the mass scale of the supersymmetric particles.

In Section 5 for completeness of this work, we summarize the defining features of G_2-MSSM models.

2 Yukawa couplings and trilinear terms

One goal of this analysis is to set bounds on the trilinear and soft-squared masses. The general relation in supergravity theories [15] between trilinear and Yukawa couplings is

$$
\begin{aligned}
a_{\alpha\beta\gamma} &= \langle \mathcal{F}^m \rangle \left[\left\langle \frac{\partial_m K_{\mathrm{H}}}{M_{\mathrm{P}}^2} \right\rangle Y_{\alpha\beta\gamma} + \frac{\mathcal{N} \partial Y'_{\alpha\beta\gamma}}{\partial \langle h_m \rangle} \right] \\
&\quad - \langle \mathcal{F}^m \rangle \left[\left\langle \tilde{K}^{\delta\bar{\rho}} \left(\partial_m \tilde{K}_{\bar{\rho}\alpha} \right) \right\rangle Y_{\delta\beta\gamma} + (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right],
\end{aligned}
\tag{1}
$$

where $\tilde{K}_{\bar{\alpha}\beta} = \frac{\partial^2 K}{\partial C_{\bar{\alpha}}^\dagger \partial C_\beta}$ with $C_\alpha \in \{Q, u^{c\dagger}, d^{c\dagger}, L, e^{c\dagger}, H_u, H_d\}$, that is the Greek indices help to differentiate among the different chiral superfields of the theory. Greek indices with bar are related to operations on the antichiral superfields, e.g. $C^\dagger$. Here $\tilde{K}^{\gamma\bar{\delta}}$ denotes the elements of the inverse matrix. Besides, h_m are hidden-sector fields whose $\mathcal{F}$-term vacuum expectation values break SUSY, K_{H} is the part of the Kähler potential that depends only on these fields, $\partial_m = \partial/\partial h_m$ and $\partial_{\bar{m}}^* = \partial/\partial h_{\bar{m}}^*$. After taking the flat limit, the visible-sector superpotential has to be rescaled as $W'_{\mathrm{O}} = W_{\mathrm{O}} \left\langle \frac{W_{\mathrm{H}}^*}{|W_{\mathrm{H}}|} e^{\frac{1}{2M_{\mathrm{P}}^2} \sum_m |h_m|^2} \right\rangle = \mathcal{N} W_{\mathrm{O}}$, where W_{H} is the superpotential of the hidden sector and M_{P} is the reduced Planck mass. The primed quantities enter into W'_{O} and the unprimed ones into W_{O}. For simplicity, we assume a trivial matter Kähler metric $\tilde{K}_{\bar{\alpha}\beta}$. In this case the soft-squared scalar masses are proportional to the unit matrix, and the second line in Eq. (1) vanishes. However, we allow the Yukawa couplings to depend non-trivially on h_m. Consequently, the second term in Eq. (1) gives a contribution to the trilinears that is not proportional to the Yukawa matrix. In other words, what we explore here is

$$
(a^f)_{ij} = c_{ij}^f A_{\tilde{f}} Y_{ij}^f,
\tag{2}
$$

where $i, j \in \{1, 2, 3\}$ are family indices, $f \in \{u, d, e\}$, and c_{ij}^f are unknown numbers determining the non-proportionality.

It has been realized in [16] that only for $A_{\tilde{f}} = 0$ and $m_{\tilde{f}}^2 \propto \mathbb{1}$ at M_G or at the scale where the boundary conditions of the set-up are given, we can realize at low energies, near the electroweak scale M_{EW}, the Minimal Flavour Violation (MFV) condition [17]. However, even with large $A_{\tilde{f}}$, this does not imply that FCNCs cannot be under control. In fact, even in models with a light supersymmetric spectrum, family symmetries are a nice way to control dangerous FCNCs [16, 18, 19, 20, 21, 22, 23, 24, 25, 26]. For heavy scalar masses, one may expect that supersymmetric effects will mostly decouple, hence ameliorating the SUSY flavour problem. For the concrete examples to be discussed in §5.4.1, for instance, FCNCs and CP violation will be suppressed because of the hierarchy between the gaugino and the scalar masses. However, given the precision of observations especially in the kaon sector, even suppressed SUSY contributions can be relevant.

3 Most sensitive FCNC observables

The most important indirect tests that most scenarios for physics beyond the Standard Model (SM) have to face are the electroweak precision observables, the anomalous magnetic moment of the muon, FCNCs, and CP violation. For the G_2-MSSM examples we shall discuss in §5.4.1, for instance, the electroweak parameters are worked out in such a way that contributions due to the large values of Higgs masses involved in the theory are avoided. The Higgs sector behaves as an effective single doublet, with one light scalar and the other mass eigenstates heavy.

In the FCNC sector the $K^0 - \bar{K}^0$ observables ϵ and ϵ' can indeed give us a hint of ways to restrict boundary conditions of soft terms at M_G. In this section we discuss the computation of these parameters. Recall that QCD corrections are important for these observables and therefore the different scales involved in the determination of ϵ and ϵ' play an important role. In §5, where we consider specific examples, we mention other processes as well, for example, $l_i \to l_j \gamma$, $b \to s\gamma$, and D^0-$\bar{D}^0$ mixing, which are not constraining.

3.1 ϵ

The CP-violating parameter in neutral kaon mixing is defined as

$$\epsilon = \frac{\exp(i\pi/4)}{\sqrt{2}} \frac{\mathrm{Im}\langle K^0 | H_{\mathrm{eff}}^{\Delta S=2} | \bar{K}^0 \rangle}{\Delta m_K} \tag{3}$$

with $\Delta m_K = 2\,\mathrm{Re}\langle K^0 | H_{\mathrm{eff}}^{\Delta S=2} | \bar{K}^0 \rangle$, where $H_{\mathrm{eff}}^{\Delta S=2}$ is the effective Hamiltonian describing $\Delta S = 2$ transitions in the K^0-$\bar{K}^0$ system. The SM prediction and the experimental value of ϵ are [27]

$$\begin{aligned} \epsilon^{\mathrm{SM}} &= (1.91 \pm 0.30) \times 10^{-3}, \\ |\epsilon|^{\mathrm{exp}} &= (2.228 \pm 0.011) \times 10^{-3}, \end{aligned} \tag{4}$$

respectively. It is well-known that gluino interactions typically give the most relevant SUSY contributions to ϵ for general soft parameters. How important these are when the
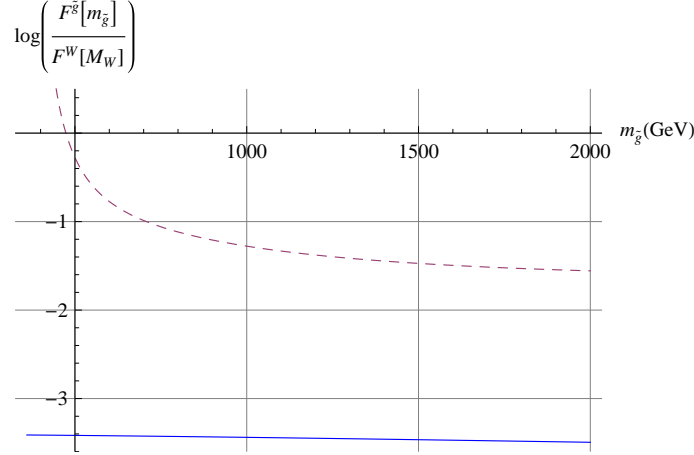
Figure 1: From top to bottom, the curves $\log[F^{\tilde{g}}(m_{\tilde{g}})/F^W(M_W)]$ for $m_{\tilde{d}} \in \{400, 10000\}$ GeV. We can see that for $m_{\tilde{d}} = 400$ GeV we need a coupling a bit more than three orders of magnitude bigger than in the SM in order to make the SUSY contribution comparable to the SM one.

scalars are heavy while the gluino remains light is an interesting question on its own. We know that the SM and gluino-sdown contributions to $\langle K^0 | H_{\text{eff}}^{\Delta S=2} | \bar{K}^0 \rangle$ are proportional to $\frac{\alpha_W^2}{4M_W^2}[(V_{td}^* V_{ts})^2 S(x_t) + (V_{cd}^* V_{cs})^2 S(x_c) + 2(V_{cd}^* V_{cs})(V_{td}^* V_{ts})S(x_t, x_c)] + \frac{\alpha_s^2}{4m_{\tilde{g}}^2} k_{\tilde{g}\tilde{d}} G_{\tilde{g}}(x_{\tilde{g}})$, where $x_t = m_t^2/M_W^2$, $x_c = m_c^2/M_W^2$ and $x_{\tilde{g}} = m_{\tilde{g}}^2/m_{\tilde{d}}^2$.[2]

In Figure 1 we have plotted $m_{\tilde{g}}$ against $\log[F^{\tilde{g}}(m_{\tilde{g}})/F^W(M_W)]$, where $F^W(M_W) = \frac{\alpha_W^2}{4M_W^2}S(x_t)$ and $F^{\tilde{g}}(m_{\tilde{g}}) = \frac{\alpha_s^2}{4m_{\tilde{g}}^2}G_{\tilde{g}}(x_{\tilde{g}})$ for two different values of the down squark mass, $m_{\tilde{d}} \in \{400, 10000\}$ GeV, from top to bottom of the graph. In the models that we study here, we can never have scalar masses as low as 400 GeV. We show the 400 GeV curve to illustrate the order of magnitude of the effective coupling entering into the CP-violating parameter ϵ that such a scalar would produce and to compare with the effect of a heavy scalar with mass above 10 TeV. From the figure we see that even if $m_{\tilde{d}} = 10$ TeV, a coupling three to four orders of magnitude bigger than the SM coupling would make the supersymmetric contribution comparable to that of the SM. Such an enhancement factor is possible due to the strong suppression of the SM contribution. For instance, $|\text{Im}\{(V_{td}V_{ts}^*)^2\}| \approx 2A^4\eta\lambda^{10} \sim 10^{-7}$, where A, η and λ are the well-known Wolfenstein parameters [28].

For the subject of this work, we are going to see, once we choose a definite flavour structure at M_G encompassing non-diagonal Yukawa and trilinear couplings, even if large mixing is present, flavour-violating effects arising from the running of soft parameters (from M_G down to M_{EW} and then to the kaon scale) are much smaller than the order of magnitude of the SM contribution and therefore flavour violation is under control in these models.

The details of the boundary conditions at M_G and the running down to the decoupling scale of scalars are given in Appendix B. We follow [30] to compute the Wilson coefficients from the $\Delta S = 2$ SUSY processes at the decoupling scale of the heavy scalars. We have

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[2]The values used here are those from [28]. The functions $S(x_i, x_j)$ are the well-known Inami-Lim functions [29] entering in the SM box contributions; $S(x)$ is listed also in Appendix A.2. $G_{\tilde{g}}(x_{\tilde{g}})$ is the loop function of the box diagram involving internal gluinos and squarks and is defined in Appendix A.2.

checked explicitly that in our scenario the gluino indeed gives the largest supersymmetric contribution. From the scale at which scalars decouple, $\mu_{\tilde{f}}$, we then follow the effective Hamiltonian approach [31].

There are two steps in the evolution from $\mu_{\tilde{f}}$ down to the kaon scale. The first step is to evolve the effective Hamiltonian to the scale where the gluinos decouple $\mu_{\tilde{g}}$ and the second step is to evolve from there down to the kaon scale. The Wilson coefficients involved in

$$\langle K^0 | H_{\text{eff}}^{\Delta S=2} | \bar{K}^0 \rangle = \sum_{i=1}^{5} C_i \langle O_i \rangle + \sum_{i=1}^{3} \tilde{C}_i \langle \tilde{O}_i \rangle \tag{5}$$

at $\mu_{\tilde{f}}$ are

$$
\begin{aligned}
C_1 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} (\delta_{\text{LL}}^d)_{12}^2 \left[24 x f_6(x) + 66 \tilde{f}(x) \right], \\
C_2 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} (\delta_{\text{RL}}^d)_{12}^2 \, 204 x f_6(x), \\
C_3 &= \frac{\alpha_s^2}{216 m_{\tilde{q}}^2} (\delta_{\text{RL}}^d)_{12}^2 \, 36 x f_6(x), \\
C_4 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} \left[(\delta_{\text{LL}}^d)_{12} (\delta_{\text{RR}}^d)_{12} \left[504 x f_6(x) - 72 \tilde{f}_6(x) \right] - (\delta_{\text{LR}}^d)_{12} (\delta_{\text{RL}}^d)_{12} \, 132 \tilde{f}_6(x) \right], \\
C_5 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2} \left[(\delta_{\text{LL}}^d)_{12} (\delta_{\text{RR}}^d)_{12} \left[24 x f_6(x) + 120 \tilde{f}_6(x) \right] - (\delta_{\text{LR}}^d)_{12} (\delta_{\text{RL}}^d)_{12} \, 180 \tilde{f}_6(x) \right], \tag{6}
\end{aligned}
$$

where the operators are given in Appendix A.1. The coefficients $\tilde{C}_i$ and operators $\tilde{O}_i$ are obtained from C_i and O_i, respectively, by interchanging L $\leftrightarrow$ R. The functions f_6 and $\tilde{f}_6$ are defined in Appendix A.2. The mass-insertion parameters are defined as usual,

$$(\delta_{\text{XY}}^d)_{ij} = \frac{(\hat{m}_{\tilde{d}\text{XY}}^2)_{ij}}{\sqrt{(\hat{m}_{\tilde{d}\text{XY}}^2)_{ii} (\hat{m}_{\tilde{d}\text{XY}}^2)_{jj}}}, \tag{7}$$

where X,Y $\in$ {L,R} and where a hat denotes a matrix in the super-CKM (SCKM) basis [32], where Yukawa couplings are diagonal.[3]

We take the results of [31] as a first approximation for the effective Hamiltonian at the kaon scale. In particular, we use the values for the low-energy Wilson coefficients given in that work. This neglects the fact that in the scenario of [31] one sfermion family is significantly lighter than the two heavy ones, while the models of our interest, to be discussed with concrete examples in §5.4.1, contain scalar masses of the same order of magnitude. In order to estimate the impact of this difference, we have calculated the running of the strong gauge coupling due to two-loop QCD corrections with and without the contributions of the first squark family from the scale where the heaviest families decouple to the gluino mass scale. The difference between the values of $g_3(m_{\tilde{g}})$ in the two cases is only about 4%, which gives us a reason to expect the change in the running of the Wilson coefficients not to be dramatic either.

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[3]That is, $Y_{\text{diag}}^f = \hat{Y}^f = U_R^f Y^f U_L^{f\dagger}$ and consequently trilinear terms are rotated as $\hat{a}^f = U_R^f a^f U_L^{f\dagger}$ and soft mass squared matrices as $\hat{m}_{\tilde{f}\text{LL}}^2 = U_L^f m_{\tilde{Q}}^2 U_L^{f\dagger}$ and $\hat{m}_{\tilde{f}\text{RR}}^2 = U_R^f m_{\tilde{f}}^2 U_R^{f\dagger}$.

3.2 ϵ'/ϵ

We consider here the most important contributions to ϵ'/ϵ,[4] namely the gluino contributions whose significance in light of the heavy scalars is the subject of this work. They come from the chromomagnetic penguin operators

$$\overline{O}_8 = \frac{g_s}{8\pi^2}m_s\bar{s}_L\sigma^{\mu\nu}t^aG^a_{\mu\nu}d_R, \quad \overline{\tilde{O}}_8 = \frac{g_s}{8\pi^2}m_s\bar{s}_R\sigma^{\mu\nu}t^aG^a_{\mu\nu}d_L, \tag{8}$$

where $G^a_{\mu\nu}$ is the gluon field strength. The corresponding Wilson coefficients C_8 and $\tilde{C}_8$ are defined as in [33].

The direct CP violation from these operators can be estimated as [34, 33, 35, 36]

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{11\sqrt{3}}{64\pi}\frac{w}{|\epsilon|\mathrm{Re}(A_0)}\frac{m_\pi^2 m_K^2}{F_\pi(m_s+m_d)}\frac{\alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}}\eta B_G\,\mathrm{Im}\left[x\left[\frac{\alpha_s\pi}{m_{\tilde{d}}^2}\right]^{-1}\left(C_8(x)-\tilde{C}_8(x)\right)\right],\tag{9}$$

where $w = \mathrm{Re}A_2/\mathrm{Re}A_0 = 0.045$ (A_i represents the amplitude for $K \to (\pi\pi)_{I=i}$), $F_\pi = 131\,\mathrm{MeV}$ is the pion decay constant, B_G represents the uncertainty in the hadronic matrix element calculation for the magnetic operator between K^0 and the 2 pion state, η represents the running effect from $m_{\tilde{g}}$ to m_c,

$$\eta = \left(\frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_t)}\right)^{2/21}\left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{2/23}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{2/25}.\tag{10}$$

The contributions from C_8 and $\tilde{C}_8$ coefficients can be decomposed into the chirality changing and conserving contributions as $x\left[\frac{\alpha_s\pi}{m_{\tilde{d}}^2}\right]^{-1}\left(C_8(x)-\tilde{C}_8(x)\right) = \Lambda_{\mathrm{LLRR}}(x) + \Lambda_g(x)$ with

$$\Lambda_g(x) = \left[(\delta^d_{\mathrm{LR}})_{12}-(\delta^d_{\mathrm{RL}})_{12}\right]x\left[-\frac{1}{3}M_1(x)-3M_2(x)\right],$$

$$\Lambda_{\mathrm{LLRR}}(x) = \left[(\delta^d_{\mathrm{LL}})_{12}-(\delta^d_{\mathrm{RR}})_{12}\right]\frac{m_s}{m_{\tilde{g}}}x\left[-\frac{1}{3}M_3(x)-3M_4(x)\right],\tag{11}$$

where the functions M_i are defined in [37, 33] and $x = m_{\tilde{g}}^2/m_{\tilde{d}}^2$.

The chirality-changing terms, for the models under consideration in this letter, show up in the down sector and we shall here consider the significant gluino contributions due to the off-diagonal a-terms which can arise from the non-proportionality between Yukawa and trilinear couplings after diagonalizing Yukawa couplings

$$(\delta^d_{\mathrm{LR}})_{12} = \frac{a^d_{12}\langle H_d\rangle}{\hat{m}^2_{\tilde{d}\mathrm{LR}}}, \quad (\delta^d_{\mathrm{RL}})_{12} = \frac{a^d_{21}\langle H_d\rangle}{\hat{m}^2_{\tilde{d}\mathrm{RL}}},\tag{12}$$

$\hat{m}^2_{\tilde{d}\mathrm{XY}}$ being the average of the two diagonal elements as in Eq. (7) which on the other hand can keep electric dipole moments (EDMs) sufficiently small [38, 39, 40].

The contributions from $(\delta^d_{\mathrm{LL}})_{12}$ and $(\delta^d_{\mathrm{RR}})_{12}$ can also be relevant if they are much bigger than $(\delta^d_{\mathrm{LR}})_{12}$ and could even overcome the enhancement factor $m_{\tilde{g}}/m_s$ that multiplies this

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[4]ϵ' being the parameter measuring the direct CP violation in the decay amplitude of $K \to 2\pi$, $\epsilon'^{i(\delta_2-\delta_0)}\mathrm{Re}[A_2](\mathrm{Re}[A_0]\mathrm{Im}[A_2]/\mathrm{Re}[A_2]-\mathrm{Im}[A_0])/(\sqrt{2}\mathrm{Re}[A_0])$, $A_Ie^{i\delta_I} = \langle\pi\pi(I)|H^{\Delta S=1}_{\mathrm{eff}}|K^0\rangle$, $I = 0,2$.

last contribution [33]. Those chirality-conserving mass insertion parameters however turn out to be more stringently constrained from Δm_K and ϵ [37, 41], and they cannot make significant contributions to ϵ' under those constraints from those indirect CP violations. We hence, in the following, discuss the effects of $(\delta^d_{\mathrm{LR,RL}})_{12}$ on ϵ'/ϵ, which can constrain the potential new physics effects on the flavour-changing interactions that may stem from the non-proportionality of trilinear and Yukawa couplings.

4 Constraints from stability of the scalar potential

Before performing the numerical analysis for the flavour violation observables, let us briefly discuss the vacuum stability bounds which constrain the flavour-violating trilinear soft terms by requiring the absence of charge or color breaking (CCB) minima and directions unbounded from below (UFB) in the scalar potential [42]. CCB and UFB constraints can become particularly important or even more stringent than those from FCNCs for large soft SUSY breaking terms, because the former are related to the ratio of scalar masses and trilinear couplings while the latter tend to decrease as the scale of SUSY breaking increases.

An undesirable deep CCB minimum appears unless the trilinear scalar couplings satisfy

$$|\hat{a}^e_{ij}|^2 \leq \left[(\hat{Y}^e_{ii})^2 + (\hat{Y}^e_{jj})^2\right]\left[(\hat{m}^2_{\tilde{e}\mathrm{LL}})_{ii} + (\hat{m}^2_{\tilde{e}\mathrm{RR}})_{jj} + m^2_{H_d} + |\mu|^2\right], \tag{13}$$

$$|\hat{a}^d_{ij}|^2 \leq \left[(\hat{Y}^d_{ii})^2 + (\hat{Y}^d_{jj})^2\right]\left[(\hat{m}^2_{\tilde{d}\mathrm{LL}})_{ii} + (\hat{m}^2_{\tilde{d}\mathrm{RR}})_{jj} + m^2_{H_d} + |\mu|^2\right], \tag{14}$$

$$|\hat{a}^u_{ij}|^2 \leq \left[(\hat{Y}^u_{ii})^2 + (\hat{Y}^u_{jj})^2\right]\left[(\hat{m}^2_{\tilde{u}\mathrm{LL}})_{ii} + (\hat{m}^2_{\tilde{u}\mathrm{RR}})_{jj} + m^2_{H_u} + |\mu|^2\right] \tag{15}$$

in the SCKM basis. Analogously to the CCB bounds, the UFB bounds for off-diagonal trilinear scalar couplings read[5]

$$|\hat{a}^e_{ij}|^2 \leq \left[(\hat{Y}^e_{ii})^2 + (\hat{Y}^e_{jj})^2\right]\left[(\hat{m}^2_{\tilde{e}\mathrm{LL}})_{ii} + (\hat{m}^2_{\tilde{e}\mathrm{RR}})_{jj} + (\hat{m}^2_{\tilde{\nu}})_{mm}\right], \tag{16}$$

$$|\hat{a}^d_{ij}|^2 \leq \left[(\hat{Y}^d_{ii})^2 + (\hat{Y}^d_{jj})^2\right]\left[(\hat{m}^2_{\tilde{d}\mathrm{LL}})_{ii} + (\hat{m}^2_{\tilde{d}\mathrm{RR}})_{jj} + (\hat{m}^2_{\tilde{\nu}})_{mm}\right], \tag{17}$$

$$|\hat{a}^u_{ij}|^2 \leq \left[(\hat{Y}^u_{ii})^2 + (\hat{Y}^u_{jj})^2\right]\left[(\hat{m}^2_{\tilde{u}\mathrm{LL}})_{ii} + (\hat{m}^2_{\tilde{u}\mathrm{RR}})_{jj} + (\hat{m}^2_{\tilde{e}\mathrm{LL}})_{pp} + (\hat{m}^2_{\tilde{e}\mathrm{RR}})_{qq}\right], \tag{18}$$

where $m \neq i, j$ and $p \neq q$. While one cannot give general predictions for the values of trilinear parameters without specifying the dependence of the Kähler potential and Yukawa couplings on the hidden-sector fields as pointed out in Eq. (1), we shall restrict the range of the off-diagonal terms $\hat{a}^f_{ij}$ by these CCB/UFB bounds when we perform the numerical studies in §5.

5 Concrete examples: G_2-MSSM models

5.1 General characteristics of the G_2-MSSM

Let us briefly overview the basic properties of the G_2-MSSM and their origin before discussing the flavour issues.

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[5]The simplified expression (16) is derived considering the $\mathcal{D}$-flat direction $\alpha^2 = |H^0_d|^2 + |\tilde{\nu}_m|^2 = |\tilde{e}_{L_i}|^2 = |\tilde{e}_{R_j}|^2$ $(m \neq i, j)$ in the limit $\alpha \gg [m^2_{H_d} + |\mu|^2 - (\hat{m}^2_{\tilde{\nu}})_{mm}]/[(\hat{Y}^e_{ii})^2 + (\hat{Y}^e_{jj})^2]$ with $\alpha^2 > |H^0_d|^2$ [42].

The starting point is a compactified M-theory, which is assumed to have the MSSM embedded in the G_2 manifold, with no extra matter, following the work of Witten [43]. The gauge group is not extended from the SM one. Supersymmetry breaking arises from the gaugino condensation mechanism, which is generic in this theory, and leads to a non-vanishing gravitino mass. The supergravity theory then allows calculating all the soft-breaking parameters in terms of the gravitino mass (detailed calculations in [8]). Then the scalar (squark and slepton and Higgs sector) masses are equal to the gravitino mass with small corrections and the trilinear factors are close to the scalar masses. The moduli Kähler and super-potentials of G_2-MSSM models are partially determined [44] G_2-holonomy Kähler potentials but the matter Kähler potentials are not [14]. In M-theory the moduli are stabilized generically because all moduli occur on an equal footing in the gauge kinetic function, and it occurs in the superpotential, so the moduli have some interactions and therefore a potential with a minimum. Their vacuum expectation values and masses can be calculated. In the G_2-MSSM, both moduli Kähler and super-potentials are basic ingredients used for the stabilization of moduli. However, matter Kähler and super-potentials do not play a role in the stabilization. Although these must also respect G_2-holonomy, the many possibilities can be reduced by studying their low-energy phenomenology.

In this respect the Kähler metric, of the G_2-MSSM considered so far, is assumed to be diagonal since the families arise at singularities on the manifold that are unlikely to overlap. However, non-trivial corrections to the off-diagonal elements of the Kähler metric may appear through higher corrections in terms of hidden sector fields. Studying effects of non-diagonal and non-universal diagonal terms phenomenologically is beyond the scope of this work, however we explore here some indirect effects by allowing deviations from the consequences of assuming a trivial Kähler metric, that is by studying effects of the non-proportionality of trilinear and Yukawa couplings.

In any case, the phenomenology of these models is characterized by a suppression of gaugino masses relative to the gravitino and the moduli masses. That scalars (squarks, sleptons, etc.) should be heavier than about 30 TeV is more general than the G_2-MSSM, depending only on the generic derivation that the moduli masses are connected to the gravitino mass, the moduli masses have a lower bound of order 30 TeV from robust cosmological arguments, and supergravity implies the scalar masses are closely equal to the gravitino mass.

We are now in a position to illustrate our aforementioned analysis using examples with a concrete UV-completion. We consider for this purpose the G_2-MSSM spectra shown in Table 1, which are characterized by heavy scalar masses of order the gravitino mass ($m_{3/2} \gtrsim O(10)\,\text{TeV}$) and a light gluino ($m_{\tilde{g}} \sim 500\,\text{GeV}$).

5.2 Typical mass spectra and couplings

The stabilization of moduli requires setting up the gravitino mass as $m_{3/2} \in (10, 100)\,\text{TeV}$ and as a result gives a definitive hierarchy of masses.

1. Heavy particles: the SUSY Higgses, the superpartners of the fermions and the Higgsinos are heavy, since their masses are related to the gravitino mass as

$$m_{\bar{\alpha}\beta}^2 \;\; = \;\; m_{3/2}^2 \delta_{\bar{\alpha}\beta},$$

Parameter	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7
$m_{3/2}$	20000	20000	20000	20000	30000	50000	30000
$\tan\beta$	3	2.65	2.65	3	3	2.5	3
μ	-11943	-13377	-13537	-10969	-10490	-34019	+17486
LSP type	Wino	Wino	Bino	Bino	Bino	Wino	Bino
$m_{\tilde{g}}$	401	449	622	492	1784	1001	596.8
$m_{\tilde{\chi}_1^0}$	145.1	155.6	189	170	473	373.4	271
$m_{\tilde{\chi}_2^0}$	153	159	214.3	181.5	702.4	397	334.2
$m_{\tilde{\chi}_1^\pm}$	145.2	155.8	214.5	181.7	702.6	373.6	334.2
$m_{\tilde{d}_L}, m_{\tilde{s}_L}$	19799	19803	19809	18785	21052	49524	29727
$m_{\tilde{b}_1}$	15342	15250	15224	14635	16783	38473	23236
$m_{\tilde{t}_1}$	9130	8779	8662	8928	11151	22887	14264
$m_{\tilde{d}_R}$	19848	19851	19845	18832	21096	49694	29794
$m_{\tilde{s}_R}$	19849	19851	19856	18832	21096	49695	29767
$m_{\tilde{t}_2}$	15342	15251	15224	14635	16783	38470	23235
m_{h_0}	116.4	114.3	114.6	116.0	115.9	115.1	114.6
$m_{H_0}, m_{A_0}, m_{H^\pm}$	24614	25846	25943	23158	25029	65690	36623

Table 1: Low-scale spectra for seven benchmark G_2-MSSM points taken from [14]. All masses are given in GeV. The other SUSY particle masses besides those shown in this table are of order the gravitino mass.

$$B, \mu \quad \sim \quad m_{3/2}. \tag{19}$$

2. Light particles: gauginos and SM particles. The gauginos become light because they are suppressed when the constraints are imposed that require a de Sitter vacuum and a small cosmological constant [13].

3. Trilinear and Yukawa couplings: the overall scale of trilinear terms is $A_{\tilde{f}} = 1.5\,m_{3/2}$ at $M_{\rm G}$ [8]. So far only particular cases of matter Kähler potentials have been studied. These studies have considered a proportionality between the Yukawa couplings and the trilinear terms.

5.3   Running of the G_2-MSSM spectra

The running of the G_2-MSSM parameters from the scale where G_2-holonomy moduli are stabilized to M_Z has been performed by some authors [8, 13, 14]. These works did not take into account the running of 3×3 Yukawa and trilinear matrices but only the running of the third-family parameters. As these effects and the moderate deviations from a proportionality between trilinear and Yukawa matrices we consider cannot have a significant influence on the masses of the superparticles, we use the results of [14] for the mass spectra of seven G_2-MSSM benchmark points, as shown in Table 1. They were calculated numerically using SOFTSUSY [45], thus taking into account the two-loop running and ensuring correct electroweak symmetry breaking as well as the absence of tachyons.

In order to calculate the low-energy mass-insertion parameters, we employ a one-loop leading-log approximation of the running of the complex Yukawa, trilinear and soft-squared

mass matrices. Numerical checks with SPheno 3.1.5 [46, 47] and SOFTSUSY 3.2.3 [45] indicate[6] that our approximation is rough but yields the correct order of magnitude. We will see that this accuracy is sufficient for the scenario studied in this work.

5.4   Example with hierarchical Yukawa couplings

We combine the G_2-MSSM spectra with the Yukawa couplings as given by the case of Fit 4 of [12] where we have updated the values of the Yukawa coefficients at the GUT scale $M_{\rm G}$,

$$
Y^d = \frac{\sqrt{2}m_b}{v\cos\beta}0.27 \begin{bmatrix} 0.0014 + 0.0007i & 0.0009 + 0.0111i & 0.13 + 0.13i \\ 0.0055 & 0.046 + 0.118i & 0.35 + 0.19i \\ 0.0018 - 0.0009i & 0.069 + 0.058i & -0.90 + 0.08i \end{bmatrix}
$$

$$
Y^u = \frac{\sqrt{2}m_t}{v\sin\beta}0.53 \begin{bmatrix} -1.58 \times 10^{-6} - 0.000017i & -0.000076 + 0.000032i & 0.0020 + 0.0020i \\ -0.00034 + 0.00024i & 0.0020 + 0.0002i & 0.011 + 0.011i \\ -0.0057 - 0.0024i & 0.0044 + 0.0115i & 0.70 + 0.71i \end{bmatrix}
$$

$$
Y^e = \frac{\sqrt{2}m_\tau}{v\cos\beta} \begin{bmatrix} 0.0014 - 0.0007i & 0.0005 - 0.0056i & 0.13 - 0.13i \\ 0.0082 & 0.023 - 0.059i & 0.18 - 0.1i \\ 0.0018 + 0.0009i & 0.035 - 0.029i & -0.99 - 0.09i \end{bmatrix}. \tag{20}
$$

As boundary conditions for the trilinear couplings, we use the relation (2) and

$$(a)\ c^f_{ij} = 1, \tag{21}$$

$$(b)\ c^f_{ij} = x^f_{ij},\ \ x^f_{ij} \in (0, \sqrt{2}) \text{ a random number and} \tag{22}$$

$$(c)\ c^f_{ij} = x^f_{ij}e^{i\varphi^f_{ij}},\ \ x^f_{ij} \in (0, \sqrt{2}),\ \varphi^f_{ij} \in (-\pi, \pi) \text{ both random numbers}, \tag{23}$$

with the exception of c^f_{33}, which is fixed to be 1 in order to preserve the aforementioned prediction for the overall scale of the trilinear couplings. All relations are valid at $M_{\rm G}$. The maximum absolute value of $|c^f| = \sqrt{2}$ is chosen to ensure that the running does not create off-diagonal elements in the soft-squared mass matrices that are larger than the diagonal elements, as explained in Appendix B.

For the case (c) above, In Table 3 we show the values of the coefficients c^d that have produced the maximum values of the flavour-violating parameters $(\delta^d_{\rm XY})_{12}$, which are listed in Table 2. For completeness we also show the values of c^u. We have chosen the matrix of coefficients $c^e = (c^d)^T$. For all SM parameters we use the values of [28].

5.4.1   CP violation in the kaon sector and vacuum stability constraints

ϵ In the G_2-MSSM cases the SUSY contribution to $\mathrm{Re}\{\langle K^0|H^{\Delta S=2}_{\rm eff}|\bar{K}^0\rangle\}$ is really small, therefore we can express $\epsilon = \epsilon^{\rm SM} + \delta\epsilon^{\rm SUSY}$ with $\delta\epsilon^{\rm SUSY} \propto \mathrm{Im}\{\langle K^0|H^{\Delta S=2}_{\rm SUSY}|\bar{K}^0\rangle\}$.

At the scale of 10 TeV, $\mathrm{Im}(\delta^d_{\rm RR})_{12}$ is the leading contribution, while the other flavour-violating parameters are at least one order of magnitude smaller. The flavour-violating parameter $(\delta^d_{\rm LR})_{12}$ involves a Yukawa coupling due to the chirality flip and is therefore suppressed for very heavy scalars. The values of $(\delta^d_{\rm RR})_{12}$ in Table 2 yield a contribution $\delta\epsilon^{\rm SUSY} \sim 10^{-6}$, safe enough in comparison to the SM contribution and the experimental limit of order 10^{-3}.

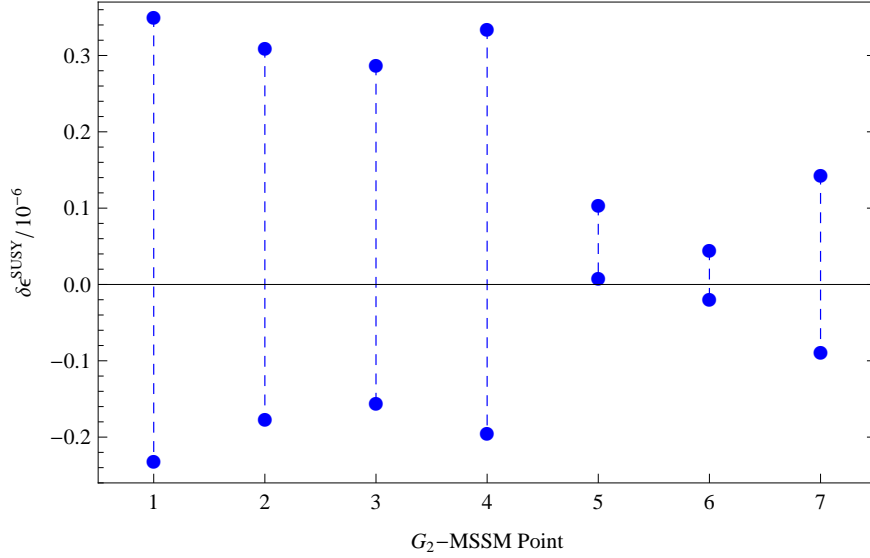[6]Neither program is completely suited for precisely the calculation required here.

Figure 2: We plot the supersymmetric contribution to ϵ for the case that trilinear terms are not proportional to Yukawa couplings but there are no new phases involved. If we add these values to the SM value together with the corresponding uncertainty, we can be in agreement with the experimental value (4), since the order of magnitude of $\delta\epsilon^{\text{SUSY}}$ is $O(10^{-7})$.

The parameters δ_{LL}^{u}, which dominate the chargino contribution to the kaon observables [48], are $O(10^{-5})$. However, we have checked that this chargino contribution is still smaller than that mediated by the gluino.

It is important to mention that in the down sector only $U_{\text{L}}^{d} \sim V_{\text{CKM}}$, while U_{R}^{d} contains large mixings. Together with the relatively large difference between the masses of $\tilde{b}_1$ and the squarks of the first and second generation, this is responsible for the difference of the off-diagonal elements in δ_{RR}^{d} and δ_{LL}^{d} in this case. We have checked the validity of the mass-insertion approximation via the vertex mixing method, analogously to the discussion for the case of $b \to s\gamma$ in section IV of [49].

In Figure 2 we plot the predictions for $\delta\epsilon^{\text{SUSY}} \approx \delta\epsilon^{\tilde{g}} + \delta\epsilon^{H^{\pm}}$ for the set-up (22), where trilinear terms are not proportional to Yukawa couplings but no new phases are involved.[7] Considering the uncertainty of ϵ^{SM}, Eq. (4), it is possible to be in agreement with the experimental value ϵ^{exp} for all the benchmark points.[8] In fact, the order of magnitude of the supersymmetric contribution, $\delta\epsilon^{\text{SUSY}}$, is the same, that is $O(10^{-7})$, as in the case with trilinear terms proportional to Yukawa couplings. In both cases, no points violate the CCB/UFB bounds.

Finally in Figure 3 we plot the maximum ranges of values of $\delta\epsilon^{\text{SUSY}}$ for the case where trilinear terms are not proportional to Yukawa couplings and where we have used Eq. (2) with the complex random coefficients c_{ij}^{f} as defined in Eq. (23). We have considered only such values for the coefficients that are allowed by the CCB/UFB constraints. This excludes about 10% of the points in the random scan. Figure 3 shows that for each G_2 point

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[7]We express ϵ as $\epsilon = \epsilon^{\text{SM}} + \delta\epsilon^{\text{SUSY}}$, $\delta\epsilon^{\text{SUSY}} = \delta\epsilon^{H^{\pm}} + \delta\epsilon^{\tilde{\chi}^{\pm}} + \delta\epsilon^{\tilde{\chi}^{0}} + \delta\epsilon^{\tilde{\chi}^{0}\tilde{g}} + \delta\epsilon^{\tilde{g}}$, where $\delta\epsilon^{\text{SUSY}}$ is the total SUSY contribution and the individual terms refer to the charged Higgs, the chargino, the neutralino, the neutralino-gluino, and the gluino contribution, respectively.

[8]For the uncertainties in the hadronic matrix element calculations, we have used the bag parameters of [50].
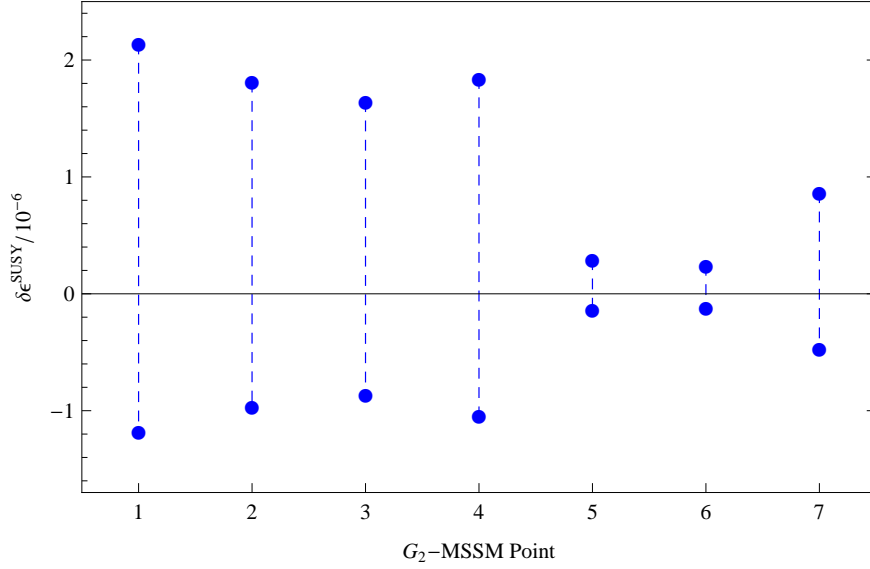
Figure 3: The same as in Figure 2, except that in this case trilinear terms are not proportional to Yukawa couplings and new phases appear, i.e., we have used Eq. (2) with complex coefficients c_{ij}^f. For each G_2-MSSM point we obtain values in agreement with the experimental value ϵ^{exp} since all contributions to $\delta\epsilon^{SUSY}$ are of $O(10^{-6})$.

all the values are in agreement with the experimental value ϵ^{exp} since all contributions to $\delta\epsilon^{\mathrm{SUSY}}$ are still at most of $O(10^{-6})$. The benchmark points 5–7 yield significantly smaller SUSY contributions due to the larger gravitino mass and consequently heavier scalars compared to points 1–4.

Re(ϵ'/ϵ) All kinds of mass insertions contribute to ϵ' [33], however those potentially large are the ones multiplied by the factor $m_{\tilde{g}}/m_s$, which are δ_{LR}^d and δ_{RL}^d, contained in the sum of the terms $C_8 O_8 + \tilde{C}_8 \tilde{O}_8 \supset H_{\mathrm{SUSY}}^{\Delta S=1}$. Due to the hierarchy of mass insertions we have found in this example, $(\delta_{\mathrm{RR}}^d)_{12} \gtrsim (\delta_{\mathrm{LL}}^d)_{12} \gg (\delta_{\mathrm{LR}}^d)_{12} \sim (\delta_{\mathrm{RL}}^d)_{12}$, we have checked if contributions from $(\delta_{\mathrm{LL}}^d)_{12}$ and $(\delta_{\mathrm{RR}}^d)_{12}$ could play an important role.

The current experimental average of ϵ'/ϵ from KTeV and NA48 is [28]

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{exp}} = (1.65 \pm 0.26) \times 10^{-3}. \tag{24}$$

With a conservative theoretical uncertainty, the SM contribution is $0 < \mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{SM}} < 3.3 \times 10^{-3}$ [51].

For the case of trilinear terms proportional to Yukawa couplings, Eq. (21), the SUSY contribution to $\mathrm{Re}(\epsilon'/\epsilon)$ is of the order 10^{-9} for all G_2-MSSM points, as expected because the off-diagonal trilinear terms generated after the running are too small. For trilinear terms not proportional to Yukawa couplings, Figure 4 shows the values of $\mathrm{Re}(\epsilon'/\epsilon)$ in the case where no phases are involved, Eq. (22), while the results with new phases, Eq. (23), are plotted in Figure 5. In all cases the SUSY contribution is smaller than 10^{-6} and thus negligible.
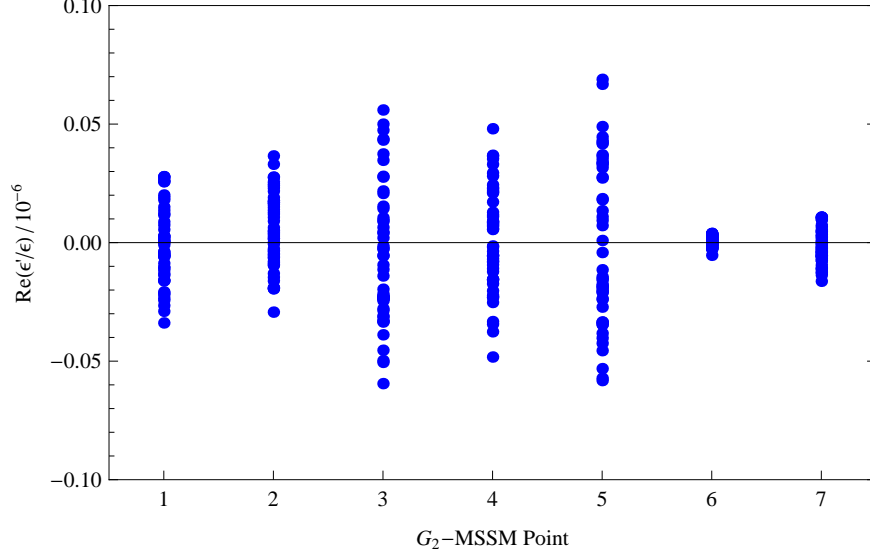
Figure 4: SUSY contribution to $\mathrm{Re}(\epsilon'/\epsilon)$ for the case where trilinear terms are not proportional to Yukawa couplings but there are no new phases involved, Eq. (22). It is far smaller than the observed value $\mathrm{Re}(\epsilon'/\epsilon) \sim 10^{-3}$. The points correspond to different random choices of the parameters determining the relation between trilinear and Yukawa couplings.
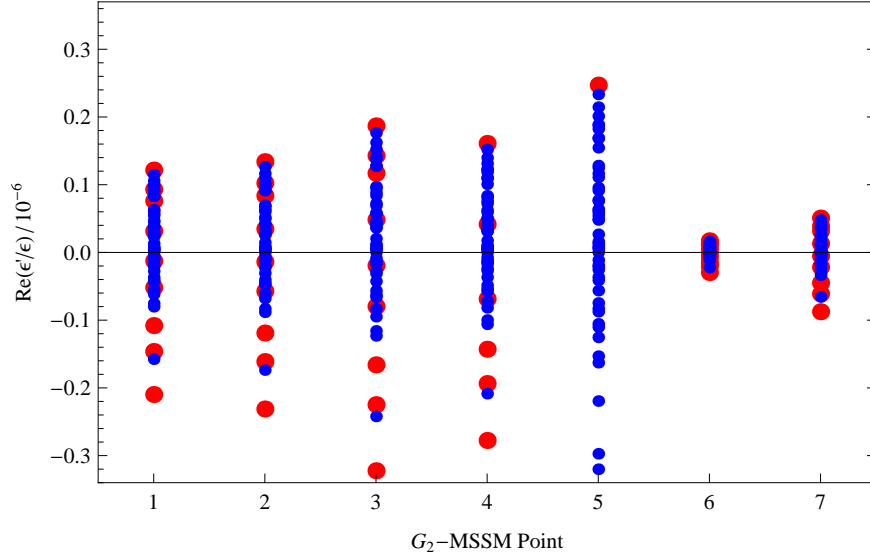

Figure 5: SUSY contribution to $\mathrm{Re}(\epsilon'/\epsilon)$ for the case where trilinear terms are not proportional to Yukawa couplings and where the parameters determining the relation between trilinear and Yukawa couplings are complex, Eq. (23). The points correspond to different random choices of these parameters. The SUSY contribution is far smaller than the observed value $\mathrm{Re}(\epsilon'/\epsilon) \sim 10^{-3}$. Some of the scanned points (larger circles, in red/light shade) violate the CCB/UFB constraints. This happens for large off-diagonal trilinear couplings, as can be seen in Eqs. (14,17). However, there are always nearby sets of parameters that give safe potentials.

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5.4.2   Further observables

Electric dipole moments.   We have discussed the effects of the off-diagonal trilinear couplings, but there are constraints on the diagonal terms as well. For instance, the experimental upper limit on the mercury EDM constrains the imaginary part of $a_{11}^{u,d}$. According to Table 3 of [52],

$$|\mathrm{Im}(\delta_{\mathrm{LR}}^{u,d})_{11}| \lesssim 10^{-6}, \tag{25}$$

if we use $m_{\tilde{q}} = 20\,\mathrm{TeV}$ and the smallest value $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 0.1$ considered in [52]. For the smaller $x \lesssim 10^{-3}$ we encounter in the G_2-MSSM, the bound may be relaxed by about an order of magnitude [6]. In all the cases analyzed, we have found that $|\mathrm{Im}(\delta_{\mathrm{LR}}^d)_{11}|$ is at most $O(10^{-8})$, while $|\mathrm{Im}(\delta_{\mathrm{LR}}^u)_{11}|$ is at most $O(10^{-7})$. An analogous constraint for $\mathrm{Im}(\delta_{\mathrm{LR}}^e)_{11}$ can be estimated from the electron EDM whose approximate contribution reads [53, 33]

$$\frac{d_e}{e} \approx \frac{\alpha_1}{4\pi} \frac{m_{\tilde{\chi}_1^0}}{m_{\tilde{l}}^2} M_1(x)\, \mathrm{Im}(\delta_{\mathrm{LR}}^e)_{11}, \tag{26}$$

where $x = m_{\tilde{\chi}_1^0}^2/m_{\tilde{l}}^2$ and the loop function $M_1(x)$ is given in Appendix A.2. We can use this rough formula to compare with the experimental upper bound of $d_e = 0.07 \times 10^{-26} e$ cm [28]. In our case, this limit requires that $\mathrm{Im}(\delta_{\mathrm{LR}}^e)_{11} \lesssim 10^{-5}$. For all the cases analyzed here, we have found that $\mathrm{Im}(\delta_{\mathrm{LR}}^e)_{11}$ is at most $O(10^{-7})$.

g − 2.   The main contributions from SUSY to the anomalous magnetic moment of the muon, $(g-2)_\mu = 2a_\mu$, come from the smuon-neutralino and the sneutrino-chargino couplings [54]. The observed value of a_μ is larger than the SM prediction by $\delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (25.9 \pm 8.1) \times 10^{-10}$ [55], so if SUSY was relevant for a_μ, the muon-neutralino and the sneutrino-chargino contributions would have to be of this order. In the case of G_2-MSSM spectra with two light gauginos and a light chargino, only the diagrams involving these particles are relevant. In this case

$$
\begin{aligned}
\delta a_\mu^{\mathrm{SUSY}} &= \delta a_\mu^{\tilde{\chi}^0 \tilde{\mu}} + \delta a_\mu^{\tilde{\chi}^\pm \tilde{\nu}}, \\
\delta a_\mu^{\tilde{\chi}^0 \tilde{\mu}} &\approx -\frac{1}{16\pi^2} \frac{m_\mu m_{\tilde{\chi}_l^0}}{m_{\tilde{\mu}_a}^2} k_{\tilde{\chi}_l^0}^a \approx -2.6 \times 10^{-10} k_{\tilde{\chi}_l^0}^a, \\
\delta a_\mu^{\tilde{\chi}^\pm \tilde{\nu}} &\approx 20 \frac{1}{16\pi^2} \frac{m_\mu m_{\tilde{\chi}_l^\pm}}{m_{\tilde{\nu}}^2} k_{\tilde{\chi}_l^\pm}^a \approx 4.4 \times 10^{-9} k_{\tilde{\chi}_l^\pm}^a.
\end{aligned}
\tag{27}
$$

The couplings $k_{\tilde{\chi}_l^0}^a$ are of the order $g_1^2 v_d(-\hat{a}_{22}^{e*} + \mu\, y_\mu \tan\beta)/((\hat{m}_{\tilde{e}\mathrm{LL}}^2)_{22} - (\hat{m}_{\tilde{e}\mathrm{RR}}^2)_{22})$, that is $O(g_1^2(\delta_{\mathrm{LL}}^e)_{12}) \sim 10^{-5}$, while $k_{\tilde{\chi}_1^\pm}^a = -K_{\tilde{\chi}_2^\pm} \approx -g_2^2 y_\mu \frac{v_d}{\mu}$ are at most $O(y_\mu 10^{-3})$. Both are too small to produce a significant contribution to $(g-2)_\mu$, and thus cannot explain the deviation from the SM expectation that has been experimentally observed [56]. We stress, however, that the SM computations are not final and the predicted SM value could change [28].

B decays   constrain the flavour-violating parameters δ_{23}^d from their gluino-sdown, neutralino-sdown contributions and δ_{23}^u from chargino-sup and charged Higgs-sup contributions. We know that these processes are sensitive to the squark mass scale and thus

expected to be quite small for the models at hand. Thus, in order to have an idea of the order of magnitude of the decay width, $\Gamma(b \to s\gamma) = \frac{m_b^5}{16\pi}|A^\gamma(\mu_b)|^2$, we can estimate the contribution to the squared amplitudes. Indeed for the cases analyzed here, the gluino-sdown contribution is the largest. This is because of the kind of Yukawa matrices we have chosen. At this scale and at leading order

$$r_{C_7} = \frac{A_{\tilde{g}}^\gamma(\mu_b)}{A_{\text{SM}}^\gamma(\mu_b)} = \frac{C_7^{\tilde{g}}(\mu_b)}{C_7^{\text{SM}}(\mu_b)} \in (-0.03, 0.03), \tag{28}$$

where the numerical value range corresponds to the range necessary to saturate the experimental 2σ region [57] and the leading-order expressions for the Wilson coefficients correspond to those of [58, 59, 60]. In the expression above, $\mu_b = 2.6$ GeV is the decay scale. Remember that we have to make the comparison at that scale because the gluino contribution follows a different QCD correction from M_W down to μ_b [61, 62]. For the analysis we follow [16]. For all the points analyzed, we have found at most $r_{C_7} = 10^{-3}$.

With a light chargino, $\tilde{\chi}_1^\pm$, and a light gluino, one may wonder if the chargino-stop and gluino-sdown loops could ever compete significantly, in cases where the contribution could be of a concern. In b decays the leading terms in the amplitudes for these two diagrams are proportional to $K^u f_1(m_{\tilde{\chi}_1^\pm}^2/m_{\tilde{t}_1}^2)$ and $K^d f_2(m_{\tilde{g}}^2/m_{\tilde{b}_1}^2)$. The mixing in the $\tilde{u}$ and $\tilde{d}$ sectors is parameterized by K^u and K^d, respectively. The loop functions f_1 and f_2 are of course different but they are of similar size whenever the ratios $m_{\tilde{\chi}_1^\pm}^2/m_{\tilde{t}_1}^2$ and $m_{\tilde{g}}^2/m_{\tilde{b}_1}^2$ are comparable. Then as long as these mass ratios are similar, a cancellation could occur or not, depending on the correlation of the mixing in the $\tilde{u}$ and $\tilde{d}$ sectors. In our case this does not occur because the mixing in these sectors is quite different.

D^0-$\bar{D}^0$ mixing is known for setting strong requirements on $(\delta_{\text{XY}}^u)_{ij}$. For a light spectrum with $m_{\tilde{q}}$ and $m_{\tilde{g}}$ around 1 TeV, the upper limits lie between 10^{-3} and 10^{-1} [63]. They are sensitive to the SUSY mass scale and become weaker for larger $m_{\tilde{q}}$. For the models discussed here, we find $(\delta_{\text{XY}}^u)_{ij} \in (10^{-5}, 10^{-6})$, so the SUSY contribution to D^0-$\bar{D}^0$ mixing is negligible.

6 Discussion

We have mainly focused on the effects of light gluinos on flavour- and CP-violating processes. In the considered scenario with heavy scalars, the SUSY contributions to flavour and CP observables are two to three orders of magnitude smaller than the SM contributions. So even if the rather crude approximations we used underestimated the SUSY contributions by an order of magnitude, the conclusion that they are negligible would still hold. Therefore, an order-of-magnitude estimate is sufficient for the present study. We leave for future work a more general study with improved accuracy and a set-up where we vary the mass scale of gluinos versus that of down squarks and sleptons.

It was beyond the scope of this letter to give a general detailed study of lepton flavour violation, partly due to the uncertainties of lepton mixing [64]. However, we have checked that the flavour-violating parameters of the G_2-MSSM models that we have analyzed are

pretty small and safe. The orders of magnitude for all cases analyzed in §5 are presented in Table 4.

In this paper, motivated by the recognition that generically compactified string and M-theories predict heavy sfermion masses and trilinear couplings ($\gtrsim 20$ TeV), we have studied in detail whether or not the decoupling effects could leave any remaining places where phenomenological issues could arise. While no concerns emerge, it is important to understand the effects on the relations between Yukawa couplings and trilinear terms, together with the improvement of hadronic uncertainties. This will help to limit the sizes and phases associated to the trilinear couplings. Most interestingly, some sets of parameters could lead to CCB or UFB potentials, but nearby sets of parameters always exist which give safe potentials. We also note that, contrary to what has been experimentally observed [56], the value we obtain for $(g-2)_\mu$ is too small to change the SM one and hence cannot provide an agreement with the experimental value at the 3σ C.L. However, the predicted SM value could change [28] due to the many uncertainties in its calculation.

Due to the very heavy squarks and sleptons characteristic for the studied scenario unacceptably large flavour- or CP-violating effects can be avoided. Therefore, in the models suggested by the compactified string/M-theories that predict heavy scalars and trilinears and assuming off-diagonal elements of Yukawas and trilinears that are not unusually large, gravity mediation of SUSY breaking does not have serious flavour and CP problems.

Let us here briefly comment on the issue of a possible tachyonic stop. While most part of flavour constraints can be relaxed by heavy first and second generation scalars, it has been pointed out that such heavy scalars could drive the squared mass of the stop, $m_{\tilde{t}}$, negative via the renormalization group evolution unless $m_{\tilde{t}} \gtrsim 7$ TeV [9, 65]. In the models of our interest, however, the scalars typically have a common mass scale of order the gravitino mass, a few tens of TeV, at the GUT scale, while the gaugino masses are typically a couple of hundred GeV at the GUT scale. Such a constraint avoids tachyonic scalars and hence is not a concern for us in this letter.

Acknowledgments

	$\lvert \mathrm{Im}(\delta^d_{\mathrm{LL}})_{12}\rvert$	$\lvert \mathrm{Im}(\delta^d_{\mathrm{RL}})_{12}\rvert$	$\lvert \mathrm{Im}(\delta^d_{\mathrm{LR}})_{12}\rvert$	$\lvert \mathrm{Im}(\delta^d_{\mathrm{RR}})_{12}\rvert$
1	2×10^{-6}	4×10^{-7}	4×10^{-7}	3×10^{-5}
2	2×10^{-6}	4×10^{-7}	4×10^{-7}	3×10^{-5}
3	2×10^{-6}	4×10^{-7}	4×10^{-7}	3×10^{-5}
4	2×10^{-6}	4×10^{-7}	4×10^{-7}	3×10^{-5}
5	1×10^{-6}	3×10^{-7}	3×10^{-7}	2×10^{-5}
6	2×10^{-6}	2×10^{-7}	2×10^{-7}	2×10^{-5}
7	2×10^{-6}	3×10^{-7}	3×10^{-7}	3×10^{-5}

Table 2: Maximum values of $\lvert \mathrm{Im}(\delta^d_{\mathrm{XY}})_{12}\rvert$ for the scanned values of the trilinear parameters for the case of Eq. (23).

c^u			c^d		
$0.28 + 0.12i$	$-0.83 - 0.76i$	$-0.31 + 1.27i$	$0.68 - 1.09i$	$0.02 - 0.16i$	$1.06 + 0.03i$
$-0.86 + 0.88i$	$-0.53 - 0.05i$	$-0.04 + 0.68i$	$-0.1 - 0.57i$	$-0.44 - 0.26i$	$-0.16 - 0.02i$
$0.01 - 0.51i$	$-0.28 - 0.38i$	1	$0.72 - 0.34i$	$-0.20 + 0.02i$	1

Table 3: Values of the coefficients c^f_{ij} of trilinear couplings, Eq. (2) and Eq. (23), for which the maximum values of $\delta\epsilon^{\mathrm{SUSY}}$ are obtained. We have analyzed a large sample of random data and from there checked that these values give the maximum flavour violating parameters for a particular G_2-point or are really close to them. The supersymmetric contributions $\delta\epsilon^{\mathrm{SUSY}}$ lie respectively for the G_2-MSSM points 1 to 7 within the following ranges: $(-1.5 \times 10^{-6}, 1.8 \times 10^{-6})$, $(-1.3 \times 10^{-6}, 1.6 \times 10^{-6})$, $(-1.1 \times 10^{-6}, 1.4 \times 10^{-6})$, $(-1.4 \times 10^{-6}, 1.7 \times 10^{-6})$, $(-1.9 \times 10^{-7}, 3.2 \times 10^{-7})$, $(-1.6 \times 10^{-7}, 2.1 \times 10^{-7})$, $(-6.1 \times 10^{-7}, 7.4 \times 10^{-7})$.

ij	$\lvert \mathrm{Im}(\delta^e_{\mathrm{LL}})_{ij}\rvert$	$\lvert \mathrm{Im}(\delta^e_{\mathrm{LR}})_{ij}\rvert$	$\lvert \mathrm{Im}(\delta^e_{\mathrm{RL}})_{ij}\rvert$	$\lvert \mathrm{Im}(\delta^e_{\mathrm{RR}})_{ij}\rvert$
12	2×10^{-7}	3×10^{-6}	5×10^{-6}	8×10^{-5}
13	1×10^{-6}	2×10^{-5}	6×10^{-5}	2×10^{-4}
23	8×10^{-6}	1×10^{-5}	8×10^{-5}	3×10^{-4}
ij	$\lvert \mathrm{Re}(\delta^e_{\mathrm{LL}})_{ij}\rvert$	$\lvert \mathrm{Re}(\delta^e_{\mathrm{LR}})_{ij}\rvert$	$\lvert \mathrm{Re}(\delta^e_{\mathrm{RL}})_{ij}\rvert$	$\lvert \mathrm{Re}(\delta^e_{\mathrm{RR}})_{ij}\rvert$
12	2×10^{-7}	3×10^{-6}	3×10^{-6}	1×10^{-4}
13	1×10^{-6}	2×10^{-5}	6×10^{-5}	3×10^{-4}
23	2×10^{-6}	1×10^{-5}	8×10^{-5}	3×10^{-4}

Table 4: Maximum values of the leptonic flavour-violating parameters for the G_2-MSSM points analyzed.

A Notation

A.1 Wilson coefficients

We follow various references [30, 31, 66] for the extraction of the effective Hamiltonian. The $\Delta S = 2$ operators involved in Eq. (5) are

$$
\begin{aligned}
O_1 &= \bar{d}^\alpha \gamma_\mu P_{\rm L} s^\alpha \bar{d}^\beta \gamma^\mu P_{\rm L} s^\beta, \quad O_2 = \bar{d}^\alpha P_{\rm L} s^\alpha \bar{d}^\beta P_{\rm L} s^\beta, \\
O_3 &= \bar{d}^\alpha P_{\rm L} s^\beta \bar{d}^\beta P_{\rm L} s^\alpha, \qquad\quad O_4 = \bar{d}^\alpha P_{\rm L} s^\alpha \bar{d}^\beta P_{\rm R} s^\beta, \\
O_5 &= \bar{d}^\alpha P_{\rm L} s^\beta \bar{d}^\beta P_{\rm R} s^\alpha,
\end{aligned}
\tag{29}
$$

where $P_{\rm L}$ and $P_{\rm R}$ are the left- and right-handed projection operators, respectively, $\tilde{O}_i = O_i({\rm L} \leftrightarrow {\rm R})$, and $\langle O_i \rangle = \langle \tilde{O}_i \rangle$.

A.2 Loop functions

We collect in this Appendix the loop functions that we have used in our analysis.

$$
S(x) = \frac{x(x^3 - 12x^2 + 6x^2 \ln x + 15x - 4)}{4(x-1)^3} \tag{30}
$$

$$
G_{\tilde{g}}(x) = \frac{(x-1)x(11x + 19) - 2x(13x + 2)\ln x - 1}{18(x-1)^3 x} \tag{31}
$$

$$
f_6(x) = \frac{6(1 + 3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5} \tag{32}
$$

$$
\tilde{f}_6(x) = \frac{6x(1 + x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5} \tag{33}
$$

$$
M_1(x) = \frac{1 + 4x - 5x^2 + 4x \ln x + 2x^2 \ln x}{2(1-x)^4} \tag{34}
$$

B Details of the running from $M_{\rm G}$ down to $\mu_{\tilde{f}}$

We take the scalar soft squared mass matrices to be proportional to the unit matrix at $M_{\rm G}$. Their running to $\mu_{\tilde{f}} \sim m_{3/2}$, the scale at which the scalars decouple, will produce off-diagonal entries. We require these off-diagonal elements to be significantly smaller than the diagonal elements, since otherwise the mass-insertion approximation would not be justified. To be concrete and conservative, let us consider $m_{3/2} = m_0 = 20$ TeV and demand

$$
(m_{\tilde{f}}^2)_{i \neq j} \ll (10 \text{ TeV})^2 \tag{35}
$$

at $\mu_{\tilde{f}}$. As we consider small values of $\tan\beta$, we can neglect the contributions of Y^d and a^d to the running. Furthermore, we consider CKM-like matrices diagonalizing Y^u, which implies $Y^{u\dagger}Y^u \sim Y^u Y^{u\dagger} \sim y_t^2 \, \text{diag}(0, 0, 1)$. Consequently, Y^u does not affect the running of the off-diagonal elements of $m_{\tilde{f}}^2$. Of course, the same is true of the terms in the renormalization group equation (RGE) of $m_{\tilde{f}}^2$ that involve gauge couplings. Thus, the only relevant terms

in the RGE are those proportional to $a^{u\dagger}a^u$ and $a^u a^{u\dagger}$. Approximating the right-hand side of the RGE by a constant value (leading-log approximation), we then obtain at $\mu_{\tilde{f}}$

$$|(m_{\tilde{Q}}^2)_{i\neq j}| \approx 0.34\,|(a^{u\dagger}a^u)_{ij}|, \tag{36}$$

$$|(m_{\tilde{u}}^2)_{i\neq j}| \approx 0.68\,|(a^u a^{u\dagger})_{ij}|, \tag{37}$$

$$|(m_{\tilde{d}}^2)_{i\neq j}| \approx 0. \tag{38}$$

Using Eqs. (2, 23), $U_{\mathrm{L}}^u \sim U_{\mathrm{R}}^u \sim V_{\mathrm{CKM}}$ as well as $Y_{\mathrm{diag}}^u \sim \mathrm{diag}(\lambda^8, \lambda^4, 1)$, where $\lambda \approx 0.23$ is the sine of the Cabibbo angle, and assuming no accidental cancellations, we obtain

$$|a^{u\dagger}a^u| \sim |a^u a^{u\dagger}| \sim A_{\tilde{f}}^2\, x_{\mathrm{max}}^2 \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \tag{39}$$

where x_{max} is the maximum value of x_{ij}^f. Thus, the strongest constraint stems from $\tilde{f} = \tilde{u}$ and $ij = 23$ in Eq. (35),

$$|(m_{\tilde{u}}^2)_{23}| \sim 0.68\, A_{\tilde{u}}^2 x_{\mathrm{max}}^2 \lambda^2 \ll (10\,\mathrm{TeV})^2. \tag{40}$$

With $A_{\tilde{f}} = 1.5\,\frac{m_0^2}{m_{3/2}} \approx 30\,\mathrm{TeV}$, this yields $x_{\mathrm{max}} \sim 1.8$. To be conservative, we have chosen $x_{\mathrm{max}} = \sqrt{2}$ for our numerical analysis.

C   Comments on MFV

Trilinear terms.   The term MFV [17] refers to scenarios where *all higher-dimensional operators, constructed from SM and fields with Yukawa interactions, are invariant under CP and under the flavour group G_F.* Here $G_F = \mathrm{SU}(3)_{q_L} \otimes \mathrm{SU}(3)_{u_R} \otimes \mathrm{SU}(3)_{d_R} \otimes \mathrm{SU}(3)_{l_L} \otimes \mathrm{SU}(3)_{e_R} \otimes \mathrm{U}(1)_B \otimes \mathrm{U}(1)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_{PQ} \otimes \mathrm{U}(1)_{e_R}$, and the Yukawa couplings are formally regarded as auxiliary fields that transform under G_F. As a consequence, *MFV requires that the dynamics of flavour violation is completely determined by the structure of the ordinary Yukawa couplings and in particular, all CP violation originates from the CKM phase.*

Because of the running of all couplings of a theory, this scenario can only be realized at one particular scale, usually a low energy scale. Starting with parameters defined at M_{G}, MFV can only be a good approximation at M_{EW}, if

$$a^f(M_{\mathrm{G}}) = Y^f(M_{\mathrm{G}})\, A^f(M_{\mathrm{G}}), \tag{41}$$

where A^f is a universal mass parameter for all families and kind of fermions that is small in comparison with other soft masses of the theory *and* if Yukawa couplings are small. This can be analyzed by studying the dependence of the RGEs of Yukawa and trilinear couplings on Y^f [16, 67].[9] If just the third family Yukawa couplings are evolved, of course the size of A^f does not matter because no off-diagonal terms are produced. With a full RG evolution of complex 3×3 Yukawa and trilinear matrices with small off-diagonal values, MFV can be emulated, albeit never reproduced, for sufficiently small values of A^f [16].

---

[9]One could start working in the basis where Y^d is diagonal and Y^u is not. Recall that it is not possible to work in a basis where both are diagonal precisely due to the CKM matrix.

Soft-squared masses The one-loop running of the soft-squared parameters $(m_{\tilde{f}}^2)_{ij}$ in the SCKM basis is governed by the β functions

$$
\begin{aligned}
\beta_{(\hat{m}_{\tilde{u}_L}^2)}^{(1)} &= U_L^u(m_{\tilde{Q}}^2 + 2m_{H_u}^2)U_L^{u\dagger}|\hat{Y}^u|^2 + U_L^u(m_{\tilde{Q}}^2 + 2m_{H_d}^2)U_L^{u\dagger}V_{\mathrm{CKM}}|\hat{Y}^d|^2V_{\mathrm{CKM}}^{\dagger} \\
&+ (|\hat{Y}^u|^2 + V_{\mathrm{CKM}}|\hat{Y}^d|V_{\mathrm{CKM}}^{\dagger})U_L^u m_{\tilde{Q}}^2 U_L^{u\dagger} + 2\hat{Y}^u(U_R^u m_{\tilde{u}}^2 U_R^{u\dagger})\hat{Y}^u \\
&+ 2V_{\mathrm{CKM}}\hat{Y}^d(U_R^d m_{\tilde{d}}^2 U_R^{d\dagger})\hat{Y}^d V_{\mathrm{CKM}}^{\dagger} + 2U_L^u a^{u\dagger}a^u U_L^{u\dagger} + 2U_L^u a^{d\dagger}a^d U_L^{u\dagger} + G_{m_{\tilde{Q}}^2}\mathbb{1} \\
\beta_{(\hat{m}_{\tilde{f}_R}^2)}^{(1)} &= U_R^f(2m_{\tilde{f}}^2 + 4m_{H_f}^2)U_R^{f\dagger}(\hat{Y}^f)^2 + 4\hat{Y}^f U_L^f m_{\tilde{Q}}^2 U_L^{f\dagger} \\
&+ 2(\hat{Y}^f)^2(U_R^f m_{\tilde{f}}^2 U_R^{f\dagger}) + 4U_R^f(a^f a^{f\dagger})U_R^{f\dagger} + G_{m_{\tilde{f}_R}^2}\mathbb{1},
\end{aligned}
\tag{42}
$$

where $f \in \{u,d\}$ and the functions $G_{m_{\tilde{f}}^2}$ contain flavour-diagonal contributions to the running involving gauge couplings and gaugino masses. Note that at an arbitrary scale $\mu \neq M_{\mathrm{G}}$, the terms which contain

$$
U_L^u(m_{\tilde{Q}}^2)U_L^{u\dagger}, \quad U_R^f(m_{\tilde{f}}^2)U_R^{f\dagger}
$$

are not diagonal because of the different running of the diagonal elements in $m_{\tilde{Q}}^2$ and $m_{\tilde{f}}^2$. Therefore, off-diagonal terms will necessarily be induced.

Recall that even if we consider only the running of the Yukawa couplings of the third family, this will produce a split in the masses of $m_{\tilde{f}}^2$. We can always choose to go to the basis where one of the Yukawa couplings is diagonal at M_{G}, but this does not guarantee diagonal soft mass-squared matrices in the SCKM basis because the fact that

$$
(m_{\tilde{f}}^2)_{11} = (m_{\tilde{f}}^2)_{22} \neq (m_{\tilde{f}}^2)_{33}
\tag{43}
$$

necessarily implies that not all of the matrices $\hat{m}_{\tilde{f}\mathrm{LL}}^2 = U_L^f m_{\tilde{Q}}^2 U_L^{f\dagger}$ and $\hat{m}_{\tilde{f}\mathrm{RR}}^2 = U_R^f m_{\tilde{f}}^2 U_R^{f\dagger}$ are diagonal. As it is known ϵ is very sensitive to this [68]. If the coupling of the particles beyond the SM was of the same order as that of the SM particles, this would push the limit on the scale of new physics entering into the $\Delta S = 2$ processes up to

$$
\Lambda_{\mathrm{MFV}\ \Delta S = 2} > 10^4 \text{ TeV}.
\tag{44}
$$

In the lepton sector, we assume heavy right-handed neutrinos that decouple close to the GUT scale. Therefore only the superpartners of right- and left-handed charged leptons as well as left-handed neutrinos can induce flavour violation. Considering the structure of fermion masses we are using, see Section 5.4 and references therein, the Yukawa coupling matrix for neutrinos is the same as that for the up-quark sector at M_{G}, therefore the flavour violation induced in this scenario is relatively small. Table 4 shows the MI parameters relevant for the observables $\ell_i \to \ell_j \gamma$.

In the G_2-MSSM case, where all the examples that are known [6] correspond to the case that trilinear couplings are proportional to Yukawa couplings, we can have a theory, depending on the choice of Yukawa couplings, for which at low energy, all flavour violation present is below the experimental bounds. It is only in this sense that we can say that we have an *ultraviolet version of MFV* but not in the sense in which MFV is defined. For the case of the relation (2) with c_{ij}^f as in Eq. (22), that is $O(1)$ real random numbers between 0

and $\sqrt{2}$, at low energies CP violating phases in addition to the CKM phase appear but also flavour violation is below experimental bounds. With c^f_{ij} as in Eq. (23), i.e., with random numbers between 0 and $\sqrt{2}$ and explicit CP phases at M_G, flavour violation is more difficult to neglect but still below the experimental bounds. This means that even with more CP phases than in the SM and large mixing present due to the choice of Yukawa couplings, after the running to low energy we obtain a theory which satisfies flavour violation constraints. Not surprisingly, the reason are the large scalar masses. What is not a trivial result of the analysis is that still bounds on the size of trilinear terms can be obtained.

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